Deep Nulling of Moving Interferers with Astronomical Phased Array Feeds

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BYU/NRAO L-Band PAFs

2006:

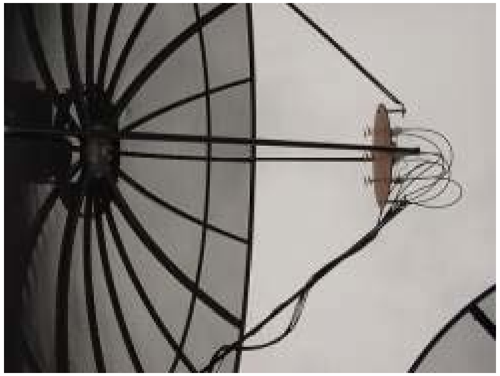
- 7 element hexagonal single-pol dipole array on 3m reflector
- Array signal processing studies
- RFI mitigation experiments

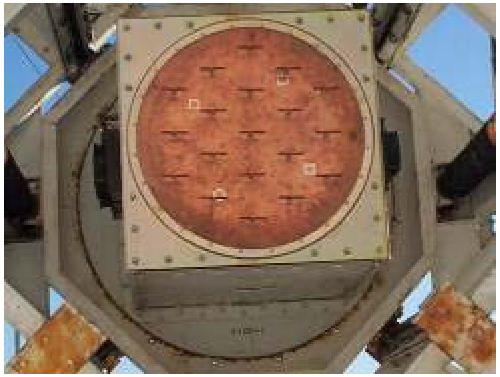
Nov. 2007:

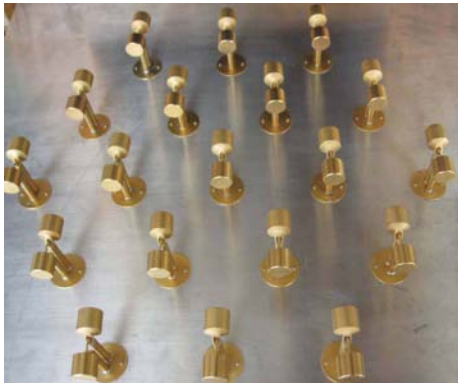
- 19 element single-pol dipole array on Green Bank 20-Meter Telescope
- Electromagnetically simple elements
- ~1 MHz instantaneous bandwidth
- Real time multichannel data acquisition
- 150 K T_{sys}
- First demonstration of on-reflector PAF

July/August 2008:

- 19 element dipole array
- 33 K LNAs (room temperature)
- 1.3 – 1.7 MHz tunable bandwidth
- *Goal: highest possible sensitivity*
- 66 K T_{sys}

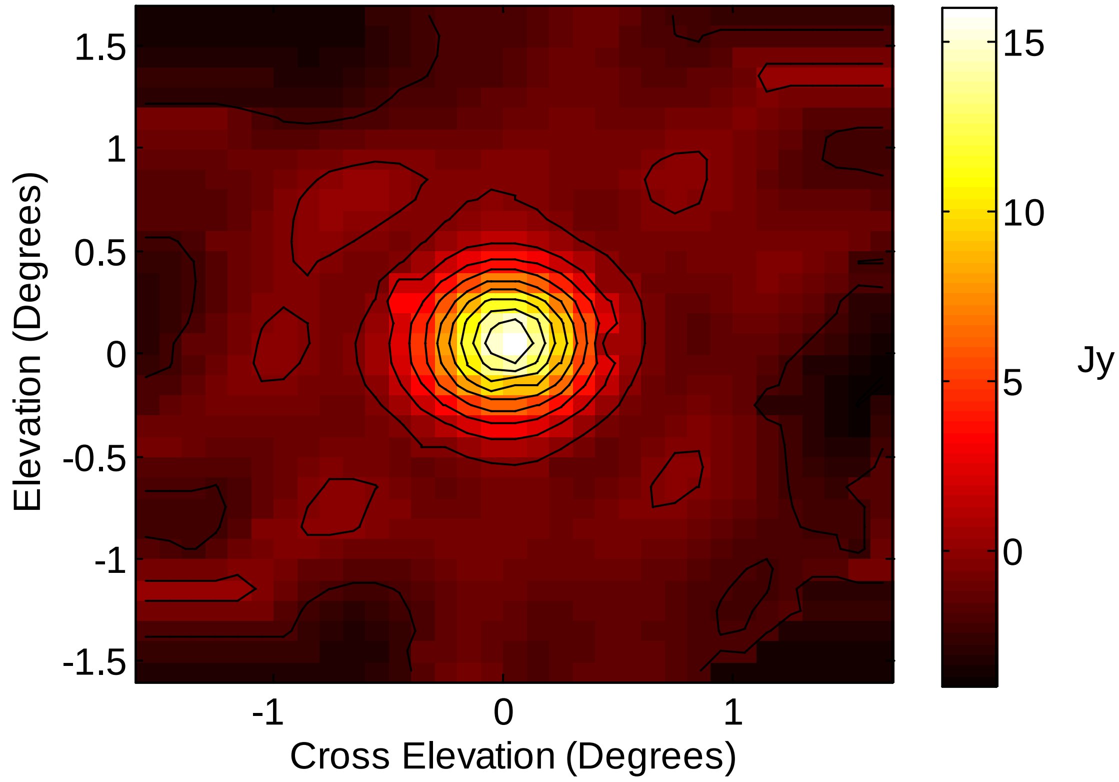




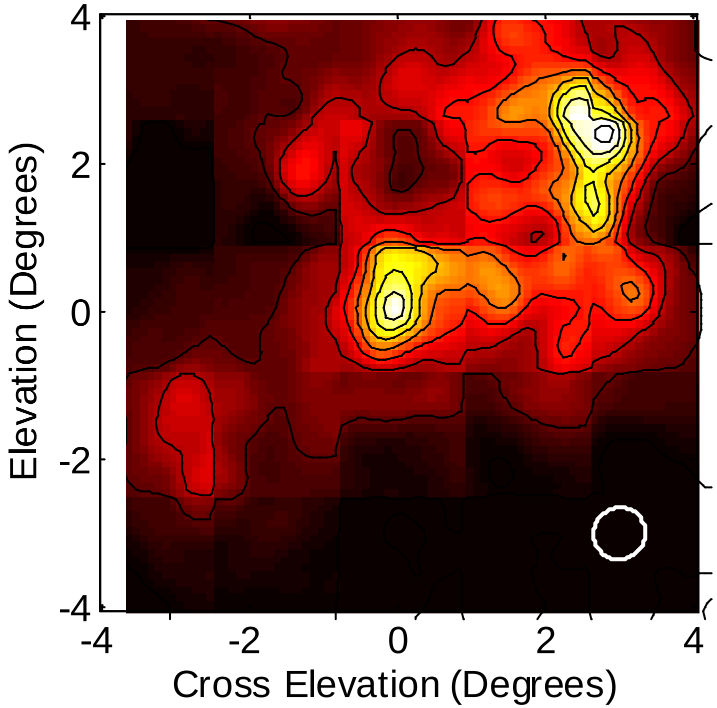


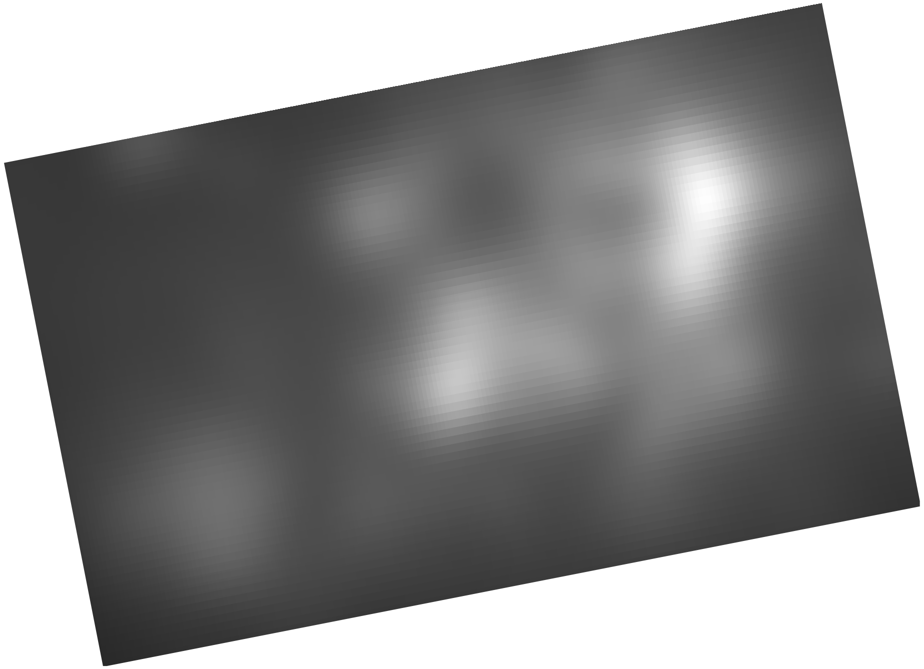
Single Pointing Image - 3C295

Source: 3C295
Flux density: 21 Jy at 1400 MHz
Observation freq. 1612 MHz
Integration time: 60 sec



Cygnus X Region at 1600 MHz





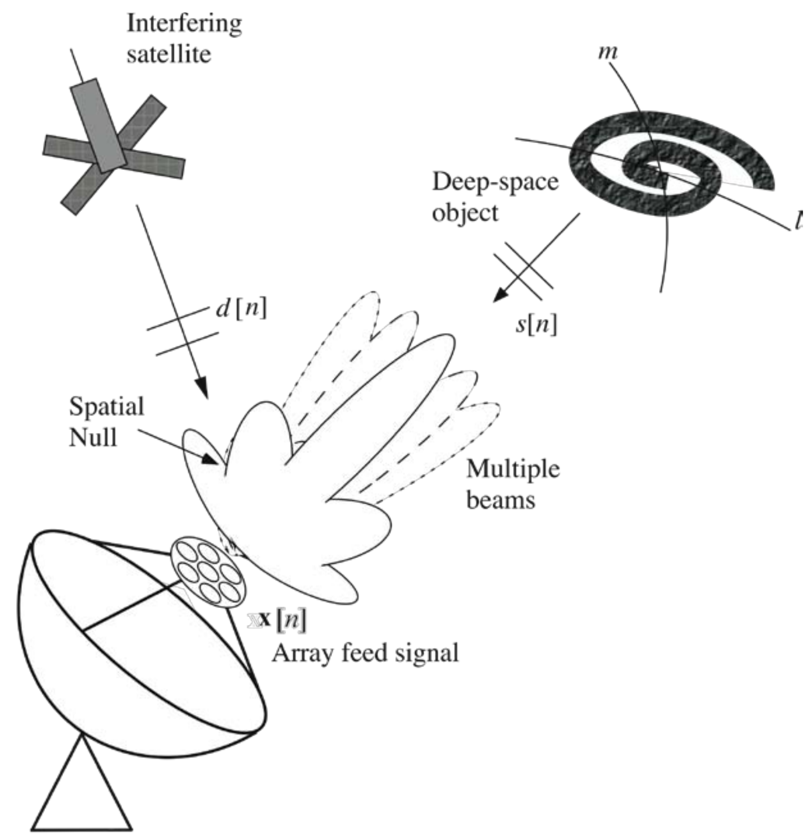
5 x 5 mosaic of PAF pointings
Circle indicates half power beamwidth
Required antenna pointings:

Single-pixel feed: ~600
PAF: 25
Imaging speedup: 24x

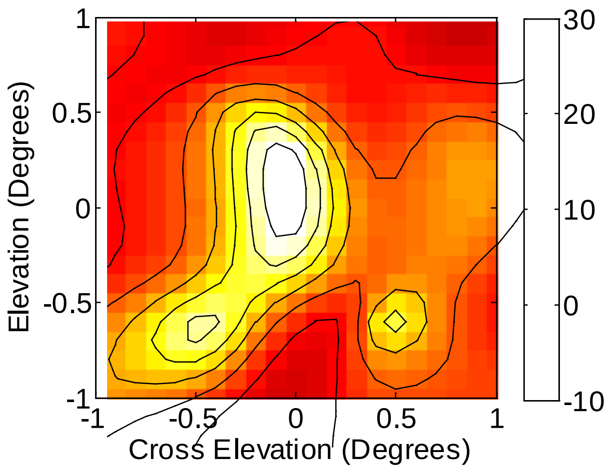
Canadian Galactic Plane Survey
Convolved to 20-Meter beamwidth

RFI Mitigation Capability

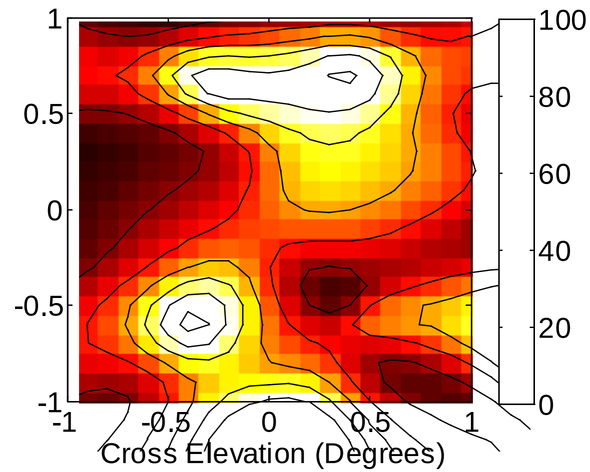
- Many degrees of freedom in array signal $\mathbf{x}[n]$ compared to horn feed.
- Adaptive beamforming algorithms place spatial nulls on RFI source.
- Covariance matrix estimate, $\hat{\mathbf{R}}_{\text{int},k}$ is updated each STI to track interference.
- Bias correction algorithm removes beamshape distortion in spectrometer output.
  [Jeffs and Warnick, *IEEE Transactions on Signal Processing*, July, 2008.]



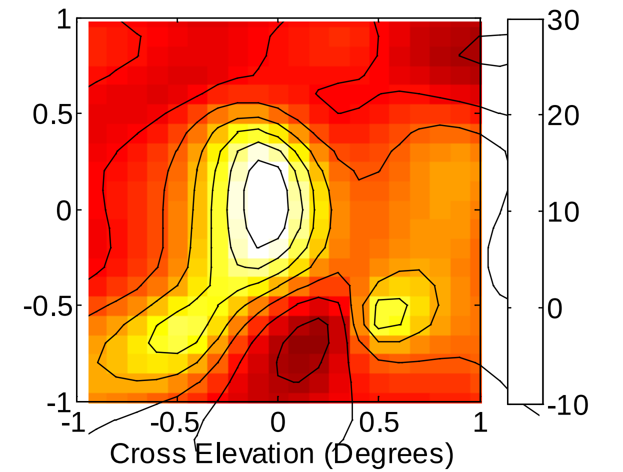
Adaptive RFI Mitigation



W3OH, no RFI



RFI corrupted image
(moving function generator
and antenna on the ground)



Adaptive spatial filtering
Subspace projection algorithm

Signal Models

- Array data vector:

$$\mathbf{x}[n] = \mathbf{a}_s s[n] + \mathbf{a}_d[n] d[n] + \boldsymbol{\eta}[n]$$

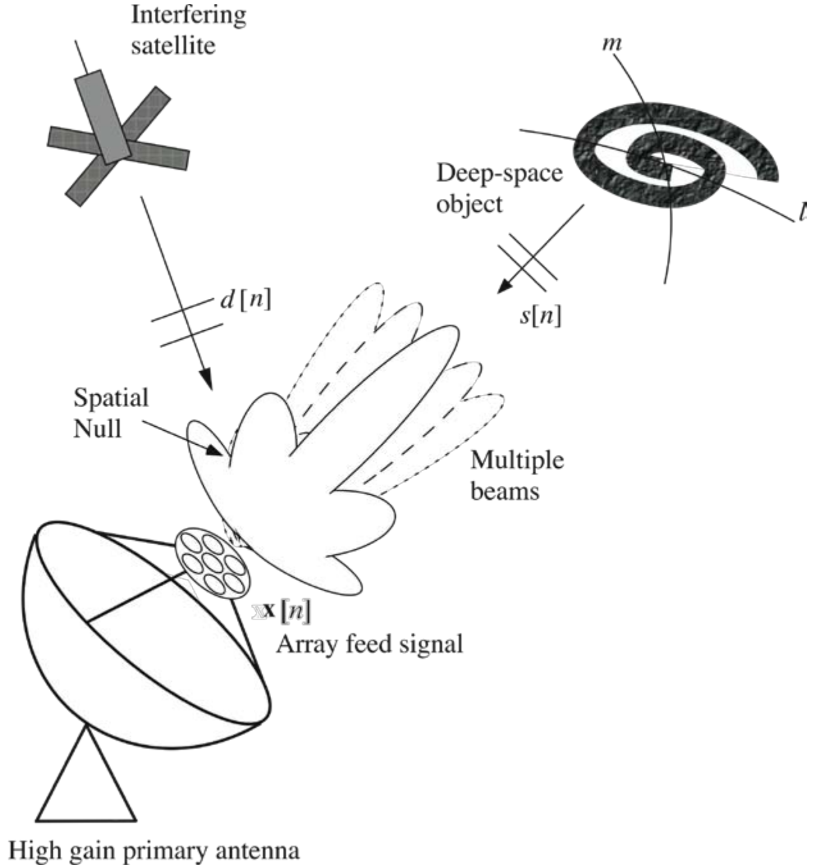
- Array covariance:
  - Time dependent due to motion in $d[n]$.
  - $\mathbf{R}[n] = E\{\mathbf{x}[n]\mathbf{x}^H[n]\}$

$$= \sigma_s^2 \mathbf{a}_s \mathbf{a}_s^H + \sigma_d^2 \mathbf{a}_d[n] \mathbf{a}_d^H[n]$$

$$= \mathbf{R}_s + \mathbf{R}_d[n] + \mathbf{R}_\eta$$

- STI covariance:

$$\mathbf{R}_k = \mathbf{R}[kN]$$



Conventional Subspace Projection (SSP)

- This zero-forcing method can place deeper nulls than max SNR, LCMV, MVDR, Weiner filter, and other array cancellers
- At *k*th STI, form an orthogonal projection matrix for the interferer(s):
  - Sample STI covariance estimate:

$$\hat{\mathbf{R}}_k = \frac{1}{N}\sum_{n=0}^{N-1}\mathbf{x}[n+kN]\mathbf{x}^H[n+kN] \quad \text{for kth STI of length } N$$

  - Partition eigenspace. Largest eigenvalues(s) correspond to interference.

$$\hat{\mathbf{R}}_k[\mathbf{U}_d \,|\, \mathbf{U}_{s+\eta}] = [\mathbf{U}_d \,|\, \mathbf{U}_{s+\eta}]\Lambda$$

  - Form projection matrix:

$$\mathbf{P}_k = \mathbf{I} - \mathbf{U}_d\mathbf{U}_d^H$$

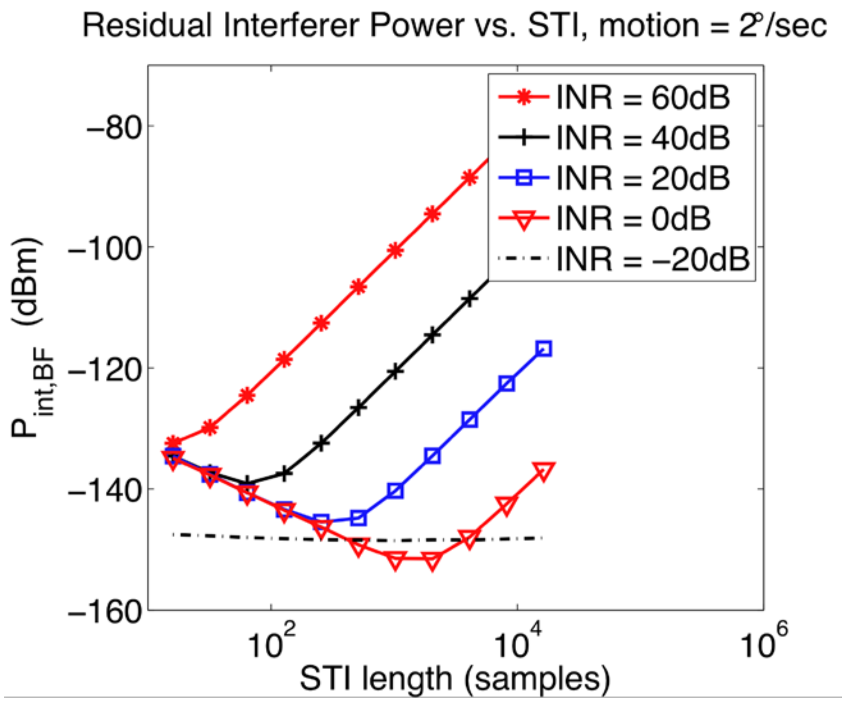
- Compute weights and beamform:

$$\mathbf{w}_{\mathrm{SSP},k} = \mathbf{P}_k\mathbf{w}_{\mathrm{nominal}}, \quad y[n] = \mathbf{w}_{\mathrm{SSP},k}^H\mathbf{x}[n], \quad k = \left\lfloor \frac{n}{N} \right\rfloor$$

Challenges for Astronomical Adaptive Array RFI Canceling

- Nulls must be deep to drive an interferer *below* the noise floor to reveal the SOI.
- Low INR interferers are hard to cancel; can't accurately estimate interference parameters.
- Conventional block update cancellation null depth is limited by:
  - Sample estimation error in $\hat{\mathbf{R}}_{d,k}$.
  - Subspace Partitioning errors: bias from $s[n]$ and correlated noise.
  - Long STIs cause subspace smearing due to motion in window.
  - But, short STIs increase sample estimation error.
  - Current methods do not exploit knowledge that interferer motion is smooth, continuous.

Challenges for Astronomical Adaptive Array RFI Canceling (continued)



- Solution:
  - Fit interference $\hat{\mathbf{R}}_{d,k}$ to a matrix polynomial model over many STIs.
  - Decrease both sample estimation error and subspace smearing.
  - Better estimates of $\hat{\mathbf{R}}_{d,k}$ yield deeper cancellation nulls!

Conventional SSP Simulation Results

- 19-element PAF on 20m reflector, 0.43f/D
- 1600 MHz signal.
- Correlated spillover noise, mutual coupling, modeled 33K Ciao Wireless LNAs.
- Measured array element radiation patterns.
- Physical Optics, full 2D integration over reflector.
- Moving point interferer
  - Average element INR: -5 dB.
  - Motions rate 3.0° / sec.
  - Traverses 3 sidelobes of boresight beam.

Subspace estimation error due to sample noise, i.e. null depth with no motion.

Subspace smearing error due to motion, i.e. null depth with no sample estimation error.

Low-order Parametric Model SSP

- Fit a series of STI covariances to a polynomial that can be evaluated at arbitrary timescale
  - Beamformer weights can be updated at every time sample, not just once per STI.
  - Use entire data window to fit polynomial ➔ less sample estimation error.
  - Exploit knowledge that physical motion yields smooth progression of $\mathbf{R}_{\text{int}}[n]$.

- Use a vector polynomial to model interference covariance at any time sample:

$$\mathbf{R}_d[n] \approx \tilde{\mathbf{R}}_d(t,\mathbf{C}) = \mathbf{a}_{\text{poly}}(t,\mathbf{C})\,\mathbf{a}_{\text{poly}}^H(t,\mathbf{C})\Big|_{t=nT_s}, \quad \text{where } \mathbf{C} = [\mathbf{c}_0 \cdots \mathbf{c}_r],$$

$$\mathbf{a}_{\text{poly}}(t,\mathbf{C}) = \mathbf{c}_0 + \mathbf{c}_1 t + \cdots + \mathbf{c}_r t^r, \quad \text{and } T_s = \text{ sample period}$$

- Minimize the squared error between STI sample covariances and the polynomial model:

$$\mathbf{C}_{\text{LS}} = \arg\min_{\mathbf{C}} \sum_{k=1}^{K} \left\| \hat{\mathbf{R}}_k - \tilde{\mathbf{R}}_d(t_k,\mathbf{C}) \right\|_F^2, \quad \text{where } t_k = kNT_s$$

Eigenvector Least Squares Fit (ELS)

- Find principal eigenvector for each STI

$$\hat{\mathbf{R}}_k \mathbf{u}_k = \lambda_{\max} \mathbf{u}_k$$

- Correct bulk phase ambiguity per STI by enforcing a smoothness constraint.
  - Phase shift each successive eigenvector so it is aligned as closely to the previous one as possible:

$$\alpha_k = \frac{\mathbf{u}_k^H \mathbf{u}_{k-1}}{\left\|\mathbf{u}_k\right\|^2} \qquad \mathbf{u}_{\mathrm{smooth},k} = \frac{\alpha_k \mathbf{u}_k}{\left\|\alpha_k \mathbf{u}_k\right\|}$$

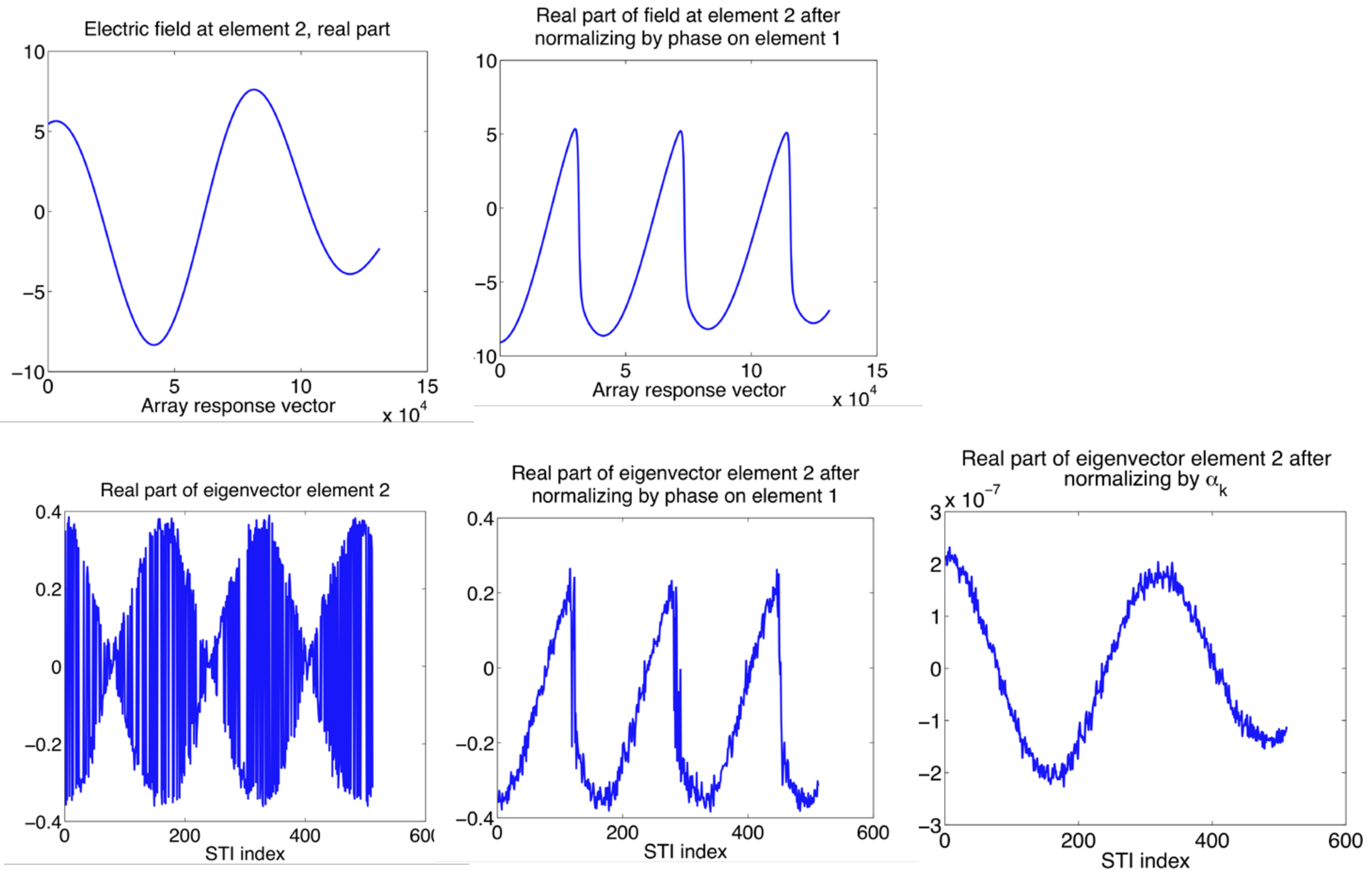
- Closed-form solution for polynomial coefficients

$$\mathbf{C}_{\mathrm{ELS}} = (\mathbf{T}^T \mathbf{T})^{-1} \mathbf{T}^T \mathbf{U},$$

$$\mathbf{t} = [t_0 \ldots \; t_{K-1}]^T, \quad \mathbf{T} = [\mathbf{t}^r \; \mathbf{t}^{r-1} \ldots \; \mathbf{t}^0]$$

$$\mathbf{U} = [\mathbf{u}_0 \ldots \; \mathbf{u}_{K-1}]$$

Need for Phase Constraint



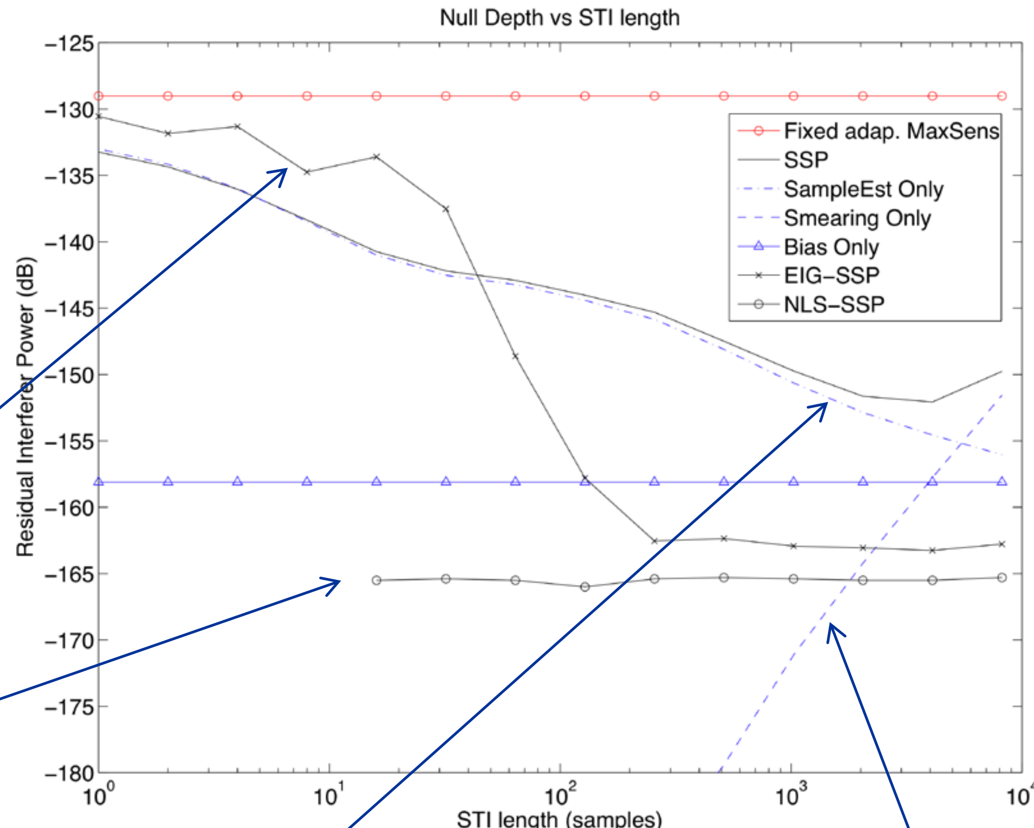
Nonlinear Least Squares Fit (NLS)

$$\mathbf{C}_{\mathrm{NLS}} = \arg\min_{\mathbf{C}} \sum_{k=1}^{K} \left\| \hat{\mathbf{R}}_k - \tilde{\mathbf{R}}_d(t_k, \mathbf{C}) \right\|_F^2 , \quad \text{where } \ t_k = kNT_s$$

- Solve with MATLAB lsqnonlin function.
- $\mathbf{a}_{\mathrm{poly}}(t_k, \mathbf{C})$ is ambiguous over the STI sequence to within one phase term:
  - Must choose this term or the optimizer will not converge.
  - Arbitrarily set first element of zero order term $[\mathbf{c}_0]_1$ to be purely real.
- Use $\mathbf{C}_{\mathrm{ELS}}$ to initialize lsqnonlin and reduce convergence time.
- NLS is slower than ELS, but produces a better fit.
- lsqnonlin convergence was improved using closed form expressions for gradients and Jacobians from the objective function.

Polynomial-augmented SSP Results (simulation)

- Polynomial order = 8.
- 13.4dB improvement over conventional SSP.
- EIG-SSP improves null depth by 11dB; NLS-SSP improves null depth by another 2.4dB.
- EIG-SSP requires enough averaging to get a good set of eigenvectors for regression.
- NLS-SSP computationally more difficult as STIs get shorter, but works over a wider range of STI lengths.



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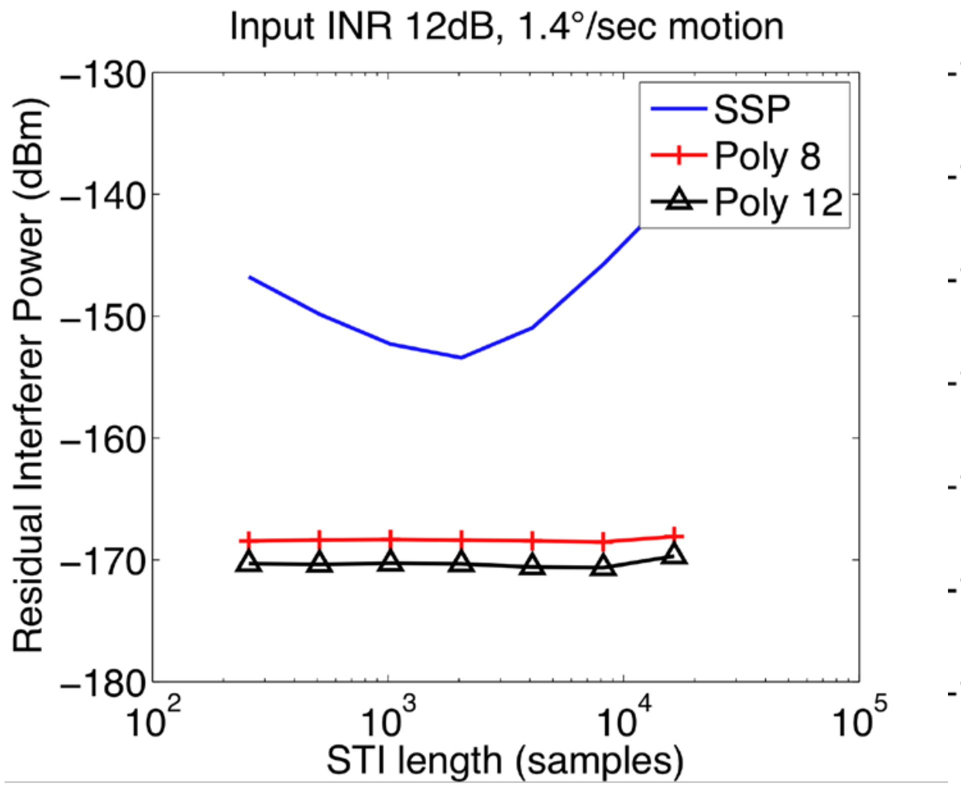
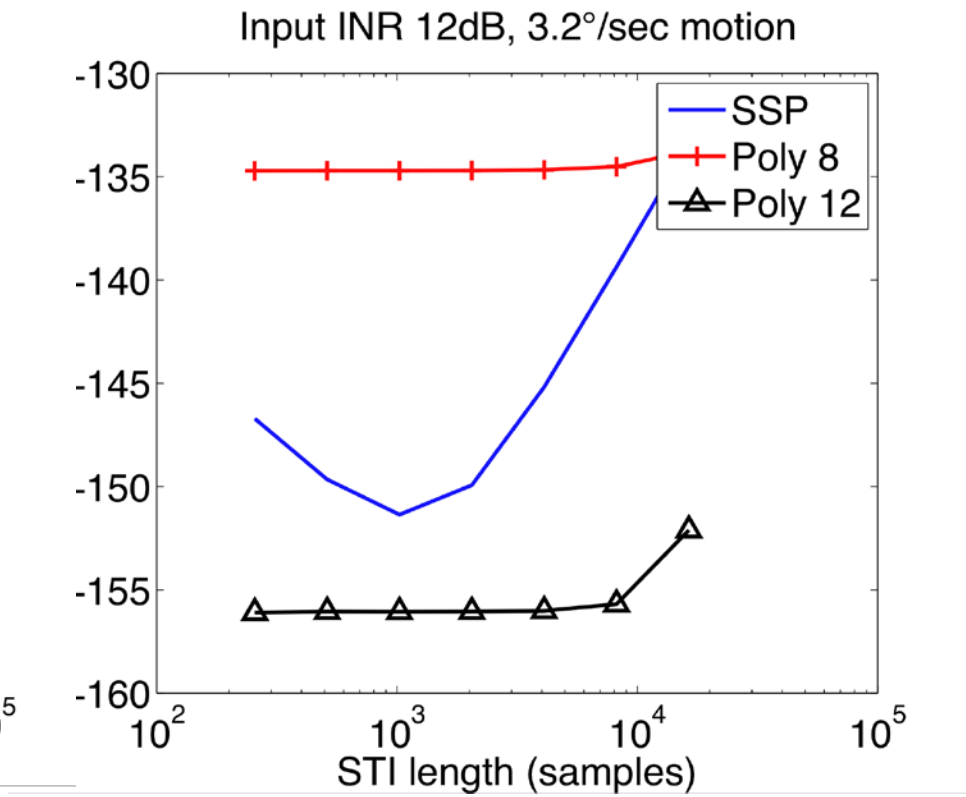
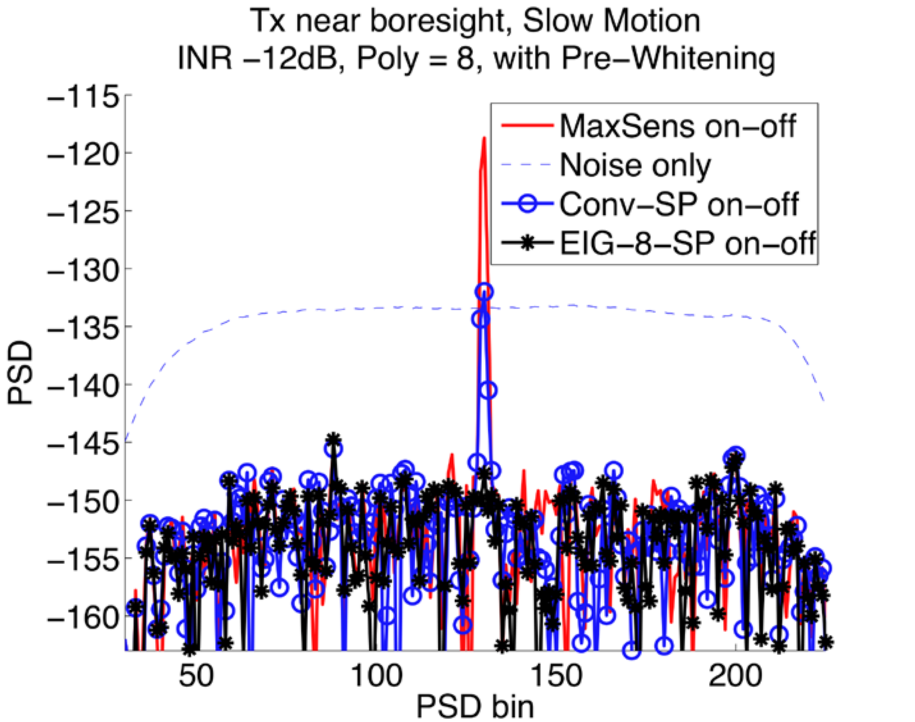
Polynomial-augmented SSP Results (simulation)

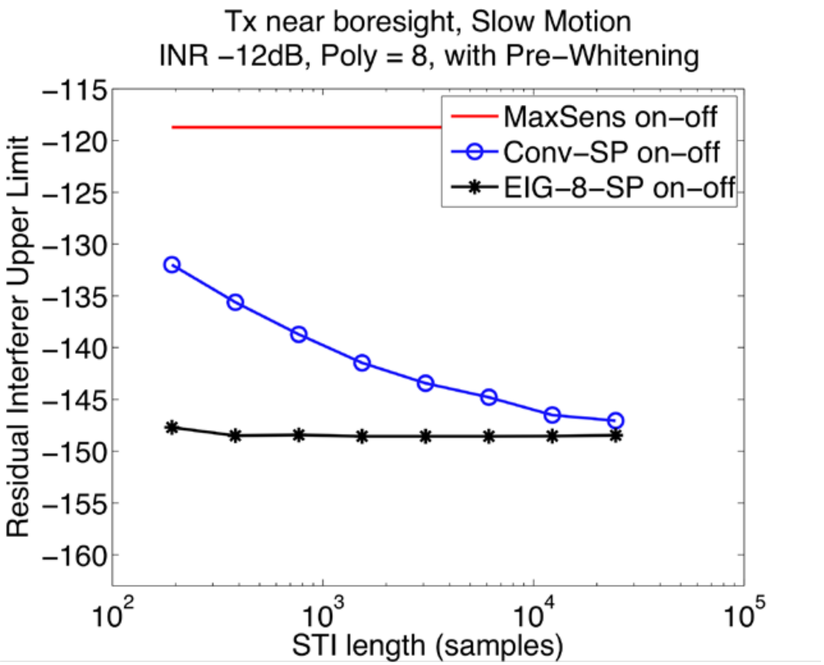




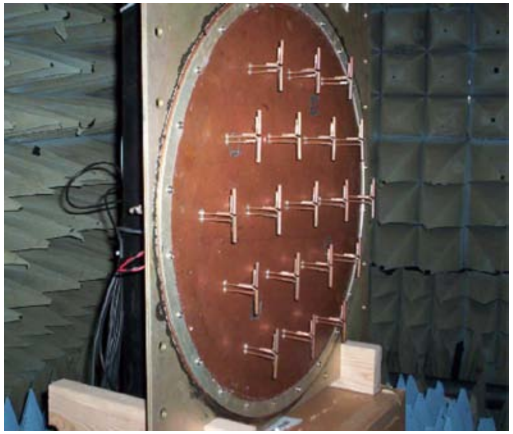
- Higher order polynomial is needed for faster motion.
- 20+ dB improvement of conventional SSP!

Real Data Results





- Interference is driven well below the noise floor!



Conclusions and Future Work

- Significant progress was demonstrated towards a truly usable adaptive PAF canceller.
- CRB Analysis to study algorithm performance:
  - Completed for white noise case:
    Mote Carlo error variance near CRB for polynomial parameters.
  - Working on full PAF correlated noise case with mutual coupling and spillover correlation.
- Need to apply polynomial assisted SSP to more real data sets.
- Will develop polynomial smoothing for the LCMV beamformer.